Test Corrections:

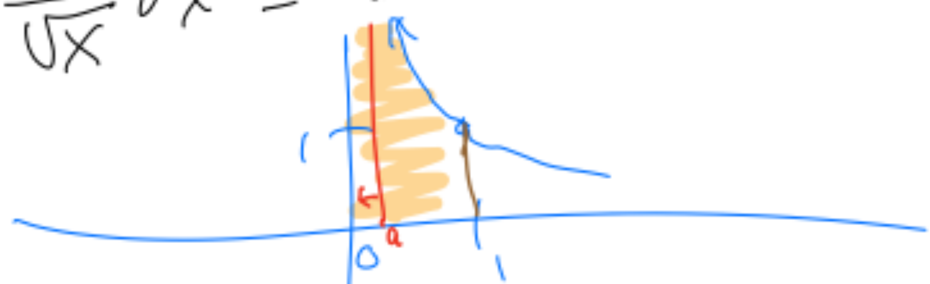
① On separate paper, redo any questions completely where points were missed.

② Explain in sentences the math mistakes made.

③ If both ① & ② are correct, get $\frac{1}{2}$ points back (on each question redone correctly) } I'm very picky.

④ You can submit multiple times to try — last submission due Fri Mar 10.

$\boxed{Ex}$ $\int_0^1 \frac{1}{\sqrt{x}}\,dx = ?$



$$= \lim_{a\to 0^+}\left(\int_a^1 \frac{1}{\sqrt{x}}\,dx\right)$$

$$\int x^{-1/2}\,dx = 2x^{1/2}+C$$

$$= \lim_{a\to 0^+} 2x^{1/2}\Big|_a^1 = \lim_{a\to 0^+}\left(2\cdot 1 - 2\cdot\sqrt{a}\right) = \boxed{2}$$

Example 2 $\displaystyle\int_0^{\pi} \sec^2(\theta)\, d\theta$

Example 3 $\displaystyle\int_0^{\pi} (s-2)^{-\frac{2}{3}}\, ds$

Example 4 $\displaystyle\int_0^{\pi} (\sec(\theta))^{\frac{2}{9}}\, d\theta$

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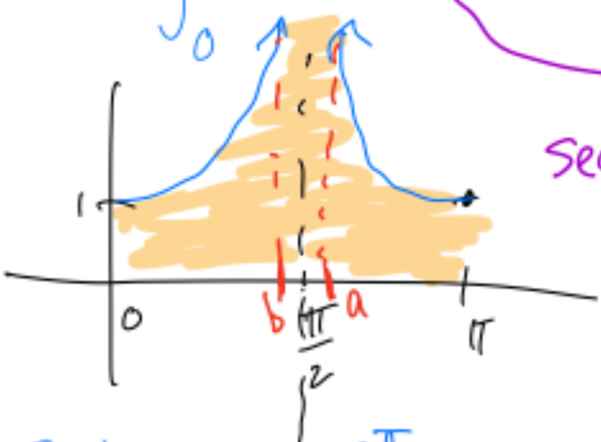
(2) $\displaystyle\int_0^{\pi} \sec^2(\theta)\, d\theta = \tan(\theta)\Big|_0^{\pi} = \tan(\pi) - \tan(0) = 0 - 0 = \boxed{0}$

wrong

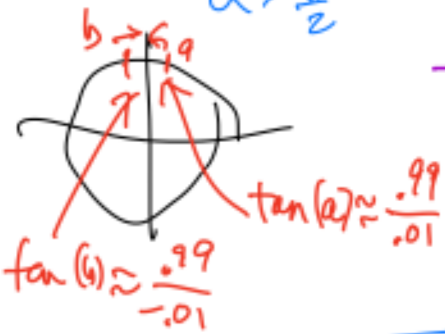
$\sec(\theta) = \dfrac{1}{\cos\theta}$ ← 0 when $\theta = \frac{\pi}{2}$



Split in two: $\displaystyle\int_0^{\pi} \sec^2(\theta)\, d\theta = \lim_{a \to \frac{\pi}{2}^-} \int_0^{a} \sec^2(\theta)\, d\theta + \lim_{b \to \frac{\pi}{2}^+} \int_b^{\pi} \sec^2(\theta)\, d\theta$

$\displaystyle = \lim_{a \to \frac{\pi}{2}^-} \left(\tan(\theta)\Big|_0^{a} \right) + \lim_{b \to \frac{\pi}{2}^+} \left(\tan\theta \right)\Big|_b^{\pi}$

$$= \lim_{a\to\frac{\pi}{2}^-}\Big(\underbrace{\tan(a)}_{\to +\infty} - \underbrace{\tan(0)}_{0}\Big) + \lim_{b\to\frac{\pi}{2}^+}\Big(\underbrace{\tan(\pi)}_{0} - \underbrace{\tan(b)}_{\to -\infty}\Big)$$



$$= \boxed{\infty}.$$

Example 3

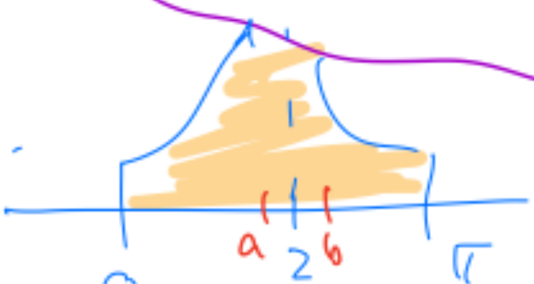
$$\int_0^\pi (s-2)^{-\frac{2}{3}}\,ds$$

$$= \int_0^\pi \frac{1}{\sqrt[3]{(s-2)^2}}\,ds$$

Not!

$$= 3(s-2)^{1/3}\Big|_0^\pi = 3(\pi-2)^{1/3} - 3(-2)^{1/3}$$

$$= \boxed{3\sqrt[3]{\pi-2} + 3\sqrt[3]{2}}$$



Let's do it the right way:

$$\int_0^\pi = \lim_{a\to 2^-}\int_0^a (s-2)^{-2/3}\,ds + \lim_{b\to 2^+}\int_b^\pi (s-2)^{-2/3}\,ds$$

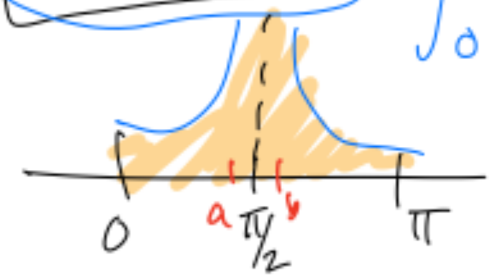
$$= \lim_{a\to 2^-} 3(s-2)^{1/3}\Big|_0^a + \lim_{b\to 2^+} 3(s-2)^{1/3}\Big|_b^\pi$$

$$= \lim_{a\to 2^-}\left(\underbrace{3(a-2)^{1/3}}_{0} - \underbrace{3(-2)^{1/3}}_{-3\sqrt[3]{2}}\right) + \lim_{b\to 2^+}\left(3(\pi-2)^{1/3} - \underbrace{3(b-2)^{1/3}}_{0}\right)$$

$3\sqrt[3]{2}$

$$= \boxed{3\sqrt[3]{2} + 3(\pi-2)^{1/3}}$$

Example 4 $\displaystyle\int_0^{\pi} (\sec(\theta))^{\frac{2}{9}}\, d\theta$



$$= \lim_{a\to \frac{\pi}{2}^-} \int_0^a (\sec\theta)^{2/9}\, d\theta$$

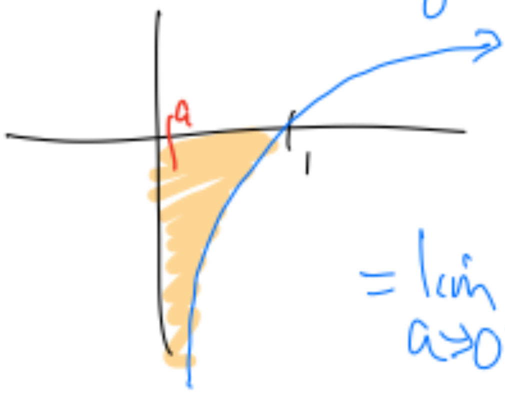
$$+ \lim_{b\to \frac{\pi}{2}^+} \int_b^{\pi} (\sec\theta)^{2/9}\, d\theta$$

$$= \lim_{a\to \frac{\pi}{2}^-} \left(\text{blech}(\theta)\right)\Big|_0^a + \lim_{b\to \frac{\pi}{2}^+} \left(\text{blech}(\theta)\right)\Big|_b^{\pi}$$

We could only find this using numerical approximations.

We need to know if the area is finite or not. Can we tell? Yes, sometimes.

Example $\int_0^1 \ln(x)\,dx$



$$= \lim_{a\to 0^+} \int_a^1 \ln(x)\,dx$$

parts $u = \ln(x)$ $du = \frac{1}{x}dx$
$dv = dx$ $v = x$

$$= \lim_{a\to 0^+} \left(x\ln(x) - \underbrace{\int dx}_{x} \right)\Big|_a^1$$

$$= \lim_{a\to 0^+} \left(x\ln(x) - x \right)\Big|_a^1$$

$$= \lim_{a\to 0^+} \left(\left(1\cdot\underbrace{\ln(1)}_{0} - 1\right) - \left(\underbrace{a}_{0}\,\underbrace{\ln(a)}_{-\infty} - \underbrace{a}_{0}\right) \right)$$

$$= -1 - \lim_{a\to 0^+} a\cdot\ln(a)$$

$0\cdot(-\infty)$ form
indeterminate form.

Other indeterminate forms:

$\pm\frac{0}{0}, \pm\frac{\infty}{\infty}, 1^\infty, 0^0, 0\cdot\infty$

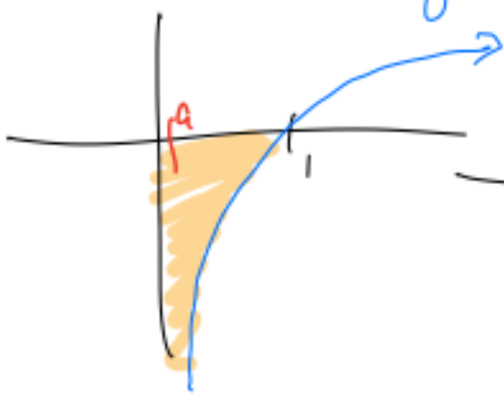
l'Hopital's Rule: If $\lim_{x\to a} f(x) = 0$, $\lim_{x\to a} g(x) = 0$,
then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$ if f & g are differentiable

↑ same formula works if it is $\frac{\infty}{\infty}$ form.

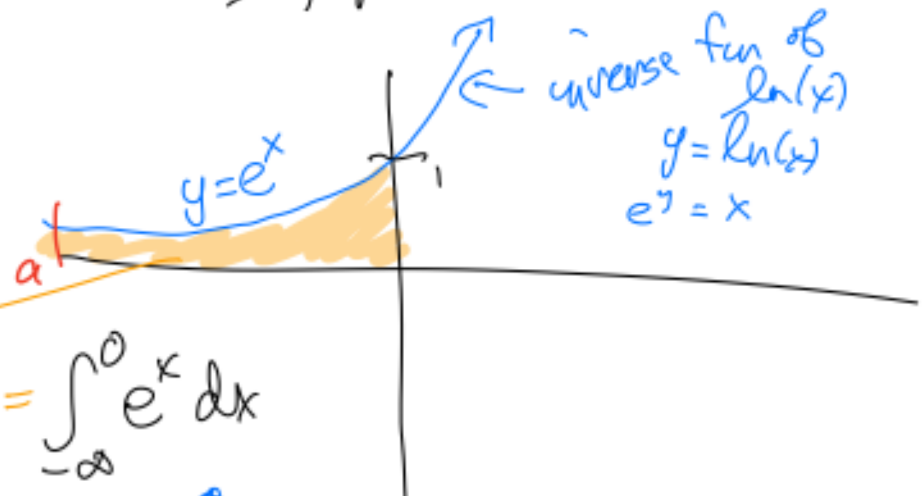
examples coming next time:

Another way to do the last integral:

$$\int_0^1 \ln(x)\,dx$$



flip x & y axes



← inverse fun of $\ln(x)$

$y = \ln(x)$

$e^y = x$

Area $= \int_{-\infty}^0 e^x\,dx$

$$= \lim_{a\to-\infty} \int_a^0 e^x\,dx$$

$$= \lim_{a\to-\infty} \left(e^x \Big|_a^0 \right) = \lim_{a\to-\infty} \left(e^0 - e^a \right)$$

$e^0 \to 1$, $e^a \to 0$

$e^{-10}, e^{-100}, e^{-1000}$

$= \boxed{1}$

$\therefore \boxed{\int_0^1 \ln(x)\,dx = -1}$

$\frac{1}{e^{10}}, \frac{1}{e^{100}}, \frac{1}{e^{1000}}$